# Quasi-Optical Ferrite Reflection Circulator

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***Abstract*—A quasi-optical Faraday rotation circulator utilizing a ferrite for microwave or millimeter-wave radiation is investigated experimentally and analyzed theoretically by a matrix formalism. Both reflection and transmission configurations at oblique incidence are examined. Numerical results in the band centered at 35 GHz are evaluated. Theory and experiment are compared over a 10–20% band. Notwithstanding the complexities resulting from oblique incidence, we find bandwidth, low loss, and isolation comparable to those of the transmission-type version now in system deployment. The principal advantage of the reflection configuration lies in the greater heat dissipation capability.**

## I. Introduction

At higher microwave and millimeter-wave frequencies, the smaller dimensions of waveguide ferrite devices severely restrict their power handling capability. To overcome this limitation, Dionne *et al.* [1] developed a quasi-optical Faraday rotator for use as either an isolator or circulator. Low-loss quasi-optical beam waveguide concepts have been implemented in a millimeter-wave system [2], and the transmission-type circulator is an integral component. Although greatly improved, the power handling capability is still limited, due to unfavorable geometry and the low thermal conductivity of the ferrite structure. The reflection version to be discussed here promises to be substantially superior in this respect, because the ferrite disc is mounted on a metal plate, which acts as a reflector and also provides an effective means of cooling, as shown in Fig. 1; in addition, only half the thickness of the ferrite is required. Whereas the transmission device normally operates with waves in normal incidence, the reflection version requires oblique incidence in order to separate the input and output beams, which introduces complexities into the analysis.

The principle of the quasi-optical circulator is illustrated in Fig. 2. The device consists of a ferrite nonreciprocal polarization rotator that furnishes Faraday rotation of 45°, with a polarizer mounted on each side. A wave entering at port 1 is rotated in a sense governed by the direction of magnetization and encounters the second polarizer, which is oriented to pass the rotated wave to port 2. The polarization of a backward-propagating wave entering port 2 is rotated 45° in the same sense, emerges cross-polarized with respect to the first polarizer, and is therefore deflected to port 3. Finally, a wave entering port 3 is deflected by the first polarizer into the rotator, emerges cross-polarized with respect to the second polarizer, and is deflected to port 4. Thus, the quasi-optical circulator is the exact analog of waveguide and other embodiments of the four-port circulator, represented schematically by the symbol in the upper left of Fig. 2. The diagram shows the transmission version of the circulator; but if it were folded at the center plane of the rotator, it would represent the reflection circulator with ports, polarizers, and rotator functioning in a similar manner.

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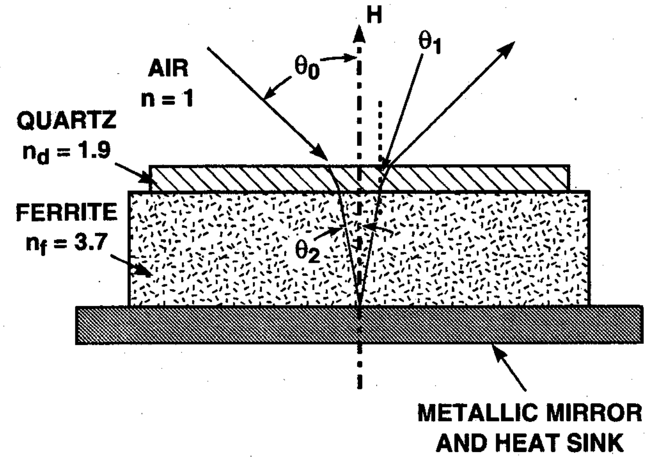


Fig. 1. Section through a reflection-type quasi-optical microwave rotator. $H$ is the magnetic field applied to the ferrite.

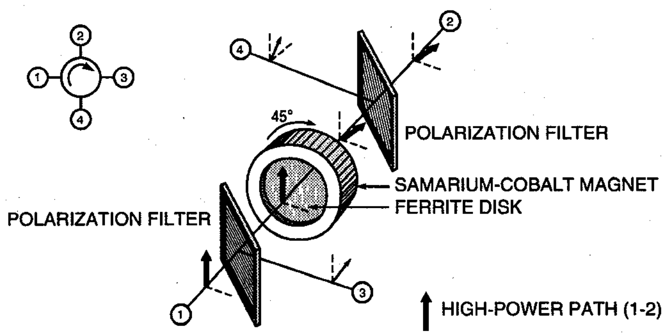


Fig. 2. A quasi-optical four-port circulator, showing a transmission-type Faraday rotator and two properly oriented polarization filters.

## II. Basic Theory

The propagation constant for microwaves propagating in a ferrite in an oblique direction with respect to the direction of magnetization is given in standard texts. For a plane wave propagating at angle $\theta$ relative to the gyrotropic axis, the propagation constants $\beta_\pm$ of the normal modes are given by [3, eq. 7-7, p. 299]

$$\frac{\beta_\pm^2}{\omega^2 \epsilon \mu_0} = \frac{1}{2(\mu \sin^2 \theta + \cos^2 \theta)} \cdot \left[2\mu + (\mu^2 - \kappa^2 - \mu)\sin^2\theta \pm \sqrt{(\mu^2 - \kappa^2 - \mu)^2 \sin^4\theta + 4\kappa^2\cos^2\theta}\right] \quad (1)$$

where $\omega$ is the radian frequency, $\epsilon$ the permittivity of the ferrite, $\mu_o$ the permeability of empty space, and $\mu$, $\kappa$ are the diagonal, off-diagonal components, respectively, of the Polder permeability tensor of the magnetized ferrite [3, eq. 4-16, p. 151]. The $\pm$ modes are elliptically polarized according to

$$\left(\frac{E_p}{E_s}\right)_+ = \left(\frac{E_s}{E_p}\right)_- = -\frac{\mu^2 - \kappa^2 - \mu}{2i\kappa}\frac{\sin^2\theta}{\cos\theta} + \sqrt{\left(\frac{\mu^2 - \kappa^2 - \mu}{2i\kappa}\right)^2 \frac{\sin^4\theta}{\cos^2\theta} - 1} \quad (2)$$

where $E_s$, $E_p$ are the component electric field amplitudes in the $s$ (perpendicular) and $p$ (parallel) polarizations with respect to the plane of incidence. Equation (1) can be simplified in the present case, since the direction of the waves in the ferrite makes a relatively small angle even for large angles of incidence, due mainly to the large dielectric constant of the ferrite ($\epsilon_r \approx 15$). Thus, for $\theta_o = 45°$ in air, $\theta_f$ in the ferrite is only about 10°. Then the Faraday rotation angle $\Theta_F$ is approximately

$$\Theta_F = \frac{1}{2c}\omega n \kappa l \cos\theta_f \quad (3)$$

where the thickness of the ferrite $T = l \cos\theta_f$, $l$ is the slant length of the beam, $n$ is the index of refraction, $c$ the velocity of light in empty space. The susceptibility $\kappa \approx \omega_M/\omega$ and $\omega_M = \gamma 4\pi M$, the angular frequency corresponding to the magnetization $4\pi M$; $\gamma$ is the gyromagnetic ratio. The above approximation includes the assumption that $\kappa << 1$. Equation (3) then simplifies to

$$\Theta_F = \frac{1}{2c} n T \omega_M. \quad (4)$$

Equation (4) still holds when we take into account the influence of the birefringence of the medium, resulting from its tensor nature, which gives rise in general to two counter-rotating elliptically polarized normal modes (1), (2) propagating in slightly different directions. Consider the ferrite slab in Fig. 3, assumed for the moment to be ideally matched at both interfaces. Considering the optical path difference of the two waves, we have

$$\Theta_F = \frac{\omega}{2c}[(n_+ l_+ - n_- l_-) - (l_+ \sin\theta_+ - l_- \sin\theta_-)\sin\theta_0]. \quad (5)$$

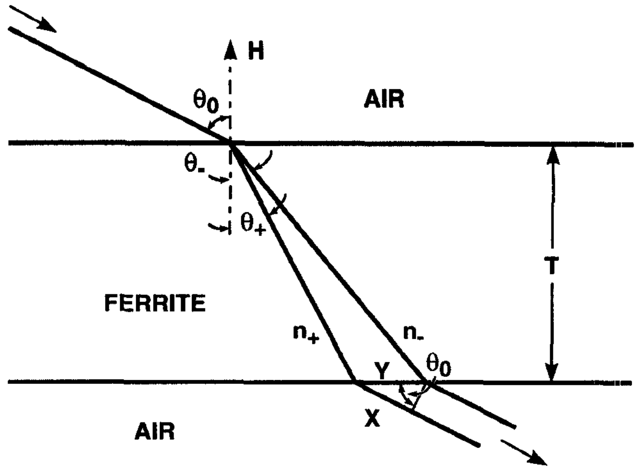


Fig. 3. Illustrating transmission through a gyrotropic ferrite slab with birefringence.

Now, $l_\pm = T/\cos\theta_\pm$, and with Snell's law $\sin\theta_o = n_\pm \sin\theta_\pm$. Thus, (5) becomes

$$\Theta_F = \frac{\omega T}{2c}(n_+ \cos\theta_+ - n_- \cos\theta_-). \quad (6)$$

This can be reduced to (4) by inserting the relation $n_\pm = n[1 \pm (\kappa/2)\cos\theta_f]$, where $n$ is a mean of $n_\pm$, and $\theta_f$ is the mean angle of refraction in the ferrite (and some small terms of higher order are neglected). Equation (4) shows that, if the ferrite layer is ideally matched, then to first order the Faraday rotation is independent of frequency and of the angle of incidence, and only depends on the magnetization and thickness of the ferrite. For proper operation of the circulator, $\Theta_F = 45°$ in the transmission-type rotator and 22.5° per traversal in the reflection type. We see that the rotator is inherently broadband, and that the ultimate limitation on bandwidth is determined by the requirement of anti-reflection plates to match the device to free space.

## III. Matrix Treatment

There are two considerations which complicate that analysis of the layered structure shown in Fig. 1: oblique incidence and the nonreciprocal tensor properties of the ferrite. The oblique direction of propagation requires that we distinguish between the $s$ and $p$ polarizations. The theory of this is well known; a 2 × 2 matrix formulation for the reflection and transmission coefficients for multilayer dielectrics is given explicitly by Yeh [4]. From that analysis, it is evident that we cannot simultaneously match waves of both $s$ and $p$ polarizations. But if a match is established for normal incidence at the center frequency, the small mismatches of the oblique waves will be of similar magnitudes for both polarizations at that frequency. In the magnetized ferrite, the Faraday rotation in effect couples the $s$ and $p$ polarizations. The polarization of the emergent wave is rotated 45° and is also slightly elliptical. In the circulator, the unwanted quadrature polarization represents a small insertion loss. In order to analyze propagation and scattering in the structure composed of dielectric quarter-wave plates, ferrite, and conductor under these conditions, it is necessary to formulate a 4 × 4 matrix representation. We have adopted the formulation of Zak *et al.* [5] with some modifications. Their system was developed for a magneto-optical investigation of thin films. Following their prescription, a product of 4 × 4 matrices for each layer and boundary is generated to form a single grand scattering matrix. From this result, a set of reflection, transmission, Faraday, and Kerr coefficients, as well as the ellipticity, can be evaluated for incident waves of $s$, $p$ or any polarization. The final grand matrices can be written as

$$S_T = A_a^{-1} \cdot A_d D_d A_d^{-1} \cdot A_f D_f A_f^{-1} \cdot A_d D_d A_d^{-1} \cdot A_a \quad (7)$$

$$S_R = A_a^{-1} \cdot A_d D_d A_d^{-1} \cdot A_f D_f A_f^{-1} \cdot A_m \quad (8)$$

where $S_T$ is for the transmission-type rotator, a product of eleven 4 × 4 matrix factors, and $S_R$ is for the reflection-type, a product of eight such factors. $A_a$ and its inverse are the "boundary" matrices in air, relating the incident and reflected electric field amplitudes $E_s$ and $E_p$ to the tangential electric and magnetic fields $E_x$, $E_y$, $H_x$, $H_y$ at the interfaces. Similarly, $A_d$, $A_d^{-1}$ are the boundary matrices for the surfaces of the dielectric, and $D_d$ is the "propagation," or transfer, matrix which transforms the $s$- and $p$-polarized waves from one boundary to the other. The matrices $A_f$, $A_f^{-1}$ relate in the same manner to the ferrite, and $D_f$ is the transfer matrix which, in the product

$$A_f D_f A_f^{-1} = M_f \quad (9)$$

incorporates the birefringence and Faraday rotation to take into account the consequences of the nonreciprocal tensor medium. Analytical expressions for these matrices were explicitly derived in [5], and are given here in Appendix I, with symbols consistent with our notation. The product $S_R$ contains the matrix $A_m$ for the conducting mirror shown in Fig. 1 (which also serves as a heat sink). Since the effective refractive index of the metal is very large, it can be eliminated, reducing the number of factors to seven—except when required for evaluation of the slight loss due to finite metallic conductivity.

## IV. Numerical Results

Calculations have been carried out for the design and performance of optimized transmission and reflection rotators, as well as for the parameters of the actual structures employed in the experimental work. Parameters chosen for the band centered at 35 GHz are: for the quartz quarter-wave plates $n_d = 1.96$, and for the ferrite $n_f = 3.83$; the magnetization $4\pi M = 1000$ Oe, which gives the ferrite thickness for the transmission device $T_f = 6.98$ mm (3.49 mm for the reflection case). The dielectric quarter-wave plate thickness $T_d = 1.17$ mm. Using these parameters in (7), we calculated the components of the scattering matrix.

In Fig. 4, the Faraday rotation angle for the transmission device is plotted from 30 to 40 GHz, for $s$- and $p$-polarized incident waves. The nominal value of the rotation parameter ($\sigma$ of [5]) was 45° at center frequency. However, due to the unavoidable mismatch required to accommodate both $s$ and $p$ polarizations equally, there is a small Fabry–Perot-type interference effect which alters this condition. Therefore, we have adjusted the thickness of the ferrite so that the mean value over the band is approximately 45°. Depending on the band selected, the mean rotation and deviation can be optimized; Fig. 4 shows that in the worse case ($p$-polarized input), the deviation is only ±0.8° over the entire ±14% band from 30 to 40 GHz (±1° deviation corresponds to a transmission loss of 0.0013 dB and a cross-polarized component 35.2 dB below that of the principal component).

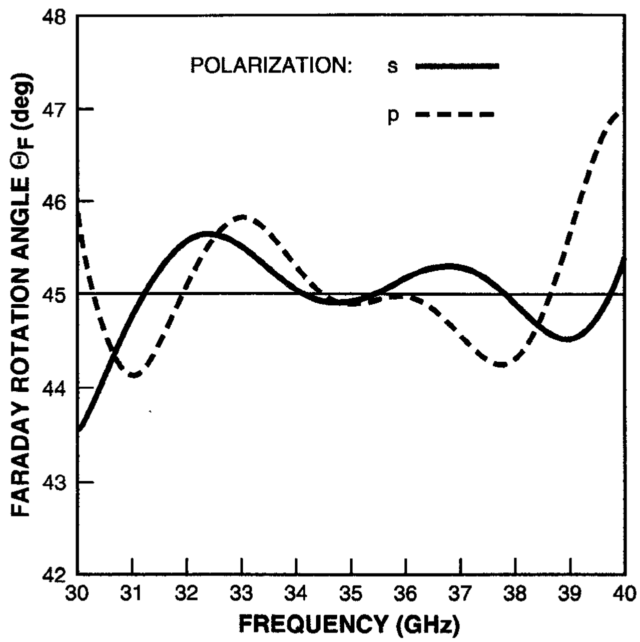


Fig. 4. Calculated frequency dependence of the Faraday rotation angle of a transmission-type rotator at angle of incidence $\theta_o = 45°$, for $s$ and $p$ polarizations of the incident wave.

Fig. 5 shows the squared magnitudes of the transmission coefficients $t_{ss}$ and $t_{ps}$ for $s$-polarized incident wave, as a function of frequency. As expected, the power is split nearly equally between the $s$ and $p$ components at 35 GHz. At the edges of the ±14% band, the combined power is reduced by ~0.5 dB, which is the contribution to the insertion loss due to the mismatch of the quarter-wave plates. This can undoubtedly be improved by application of more sophisticated antireflection techniques, such as use of multiple layers. The ellipticity was also calculated for the transmission circulator and found to be very small (below −30 db) across the band.

Calculations were also made for the reflection circulator. The Faraday rotation shown in Fig. 6 exhibits a Fabry–Perot effect which causes a variation in $\Theta_F$ of ±3° between 32 and 38 GHz. This is larger than the corresponding values for the transmission device. However, it is still small and represents a negligible contribution to the insertion loss, greatest at the band edges.

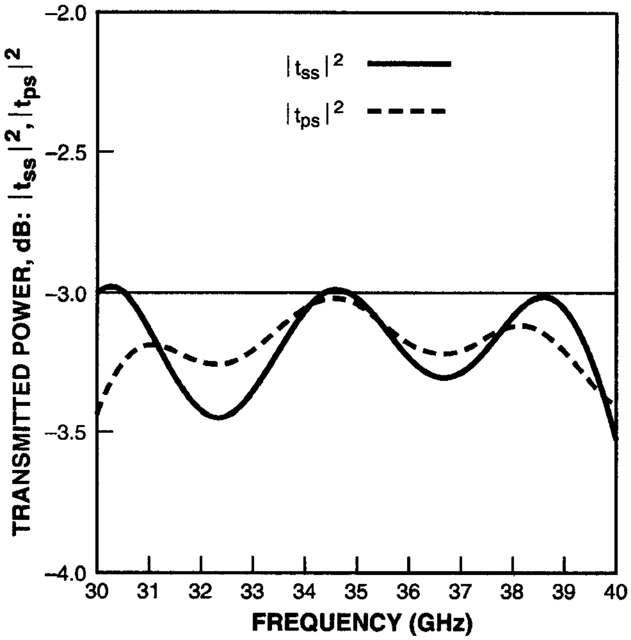


Fig. 5. Calculated frequency dependence of the transmitted power $|t_{ss}|^2$ and $|t_{ps}|^2$ of a transmission-type rotator at angle of incidence $\theta_o = 45°$.

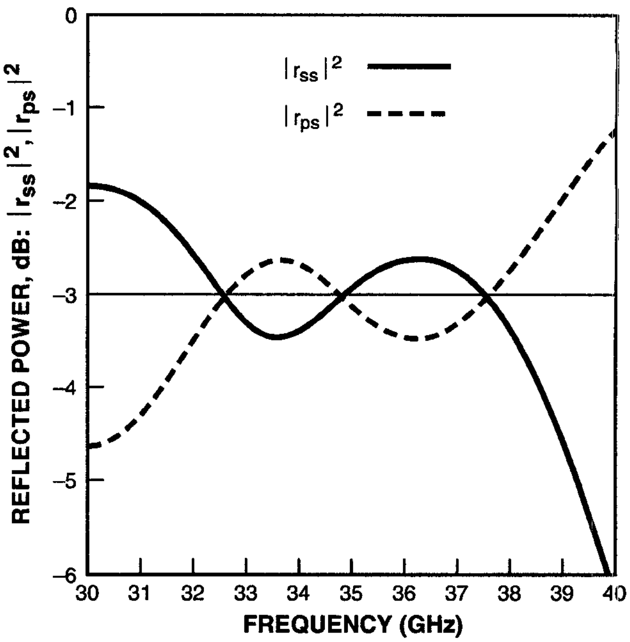


Fig. 7. Calculated frequency dependence of the reflected power $|r_{ss}|^2$ and $|r_{ps}|^2$ of a reflection-type rotator at angle of incidence $\theta_o = 45°$.

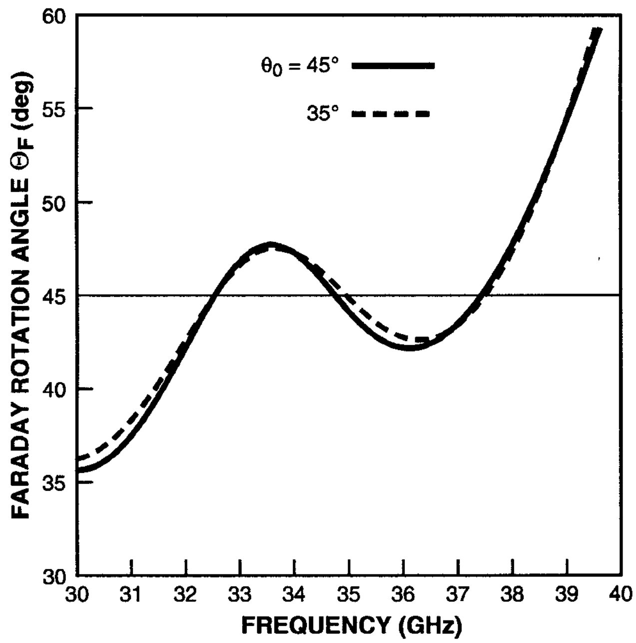


Fig. 6. Calculated frequency-dependence of Faraday rotation of a reflection-type rotator at angles of incidence $\theta_o = 35°$ and $45°$, for *s*-polarized incident wave.

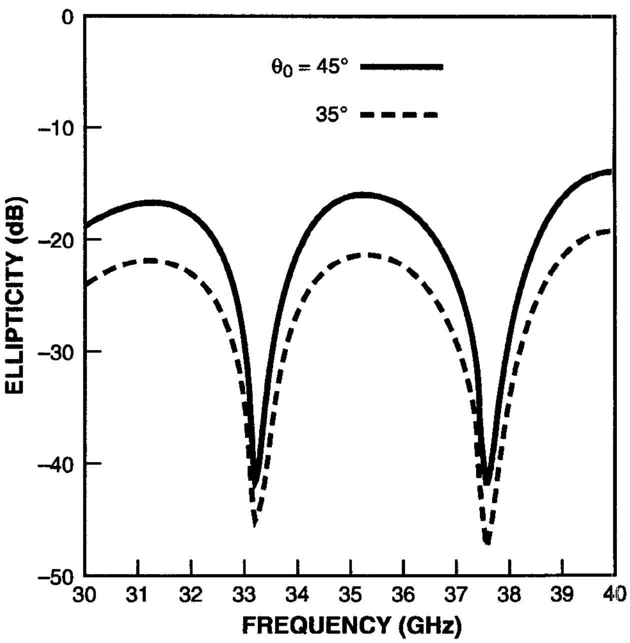


Fig. 8. Calculated frequency dependence of the ellipticity of a reflection-type rotator at angles of incidence $\theta_o = 35°$ and $45°$.

The reflection coefficient is another quantity of interest. Fig. 7 illustrates the squared magnitudes of the coefficients $r_{ss}$ and $r_{ps}$, showing the characteristic Fabry–Perot-type effect. Although the power is not equally split between the *s* and *p* components, the total adds to unity (except for minor losses in the ferrite and mirror). Performance limits on the reflection circulator are determined by the rotation and also by the ellipticity, illustrated in Fig. 8. This quantity has been calculated for several values of incidence angle and the plot shows examples at 45° and 35° incidence. It shows, as expected, that as the angle of incidence is reduced, the ellipticity decreases. The corresponding figures are 16 and 22 dB, respectively, which at worst has a slight effect on insertion loss. Due to the four-port nature of the circulator, ellipticity of the emergent wave has a negligible effect on isolation. If the contemplated application were to dictate some advantage in reducing the angle of incidence, this would have to be traded against resulting disadvantages in the system configuration.

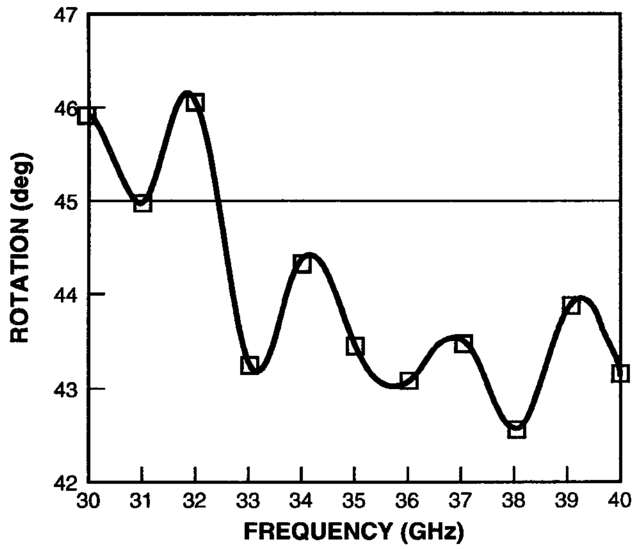


Fig. 9. Measured frequency dependence of the Faraday rotation angle of a transmission-type rotator at angle of incidence $\theta_o = 45°$.

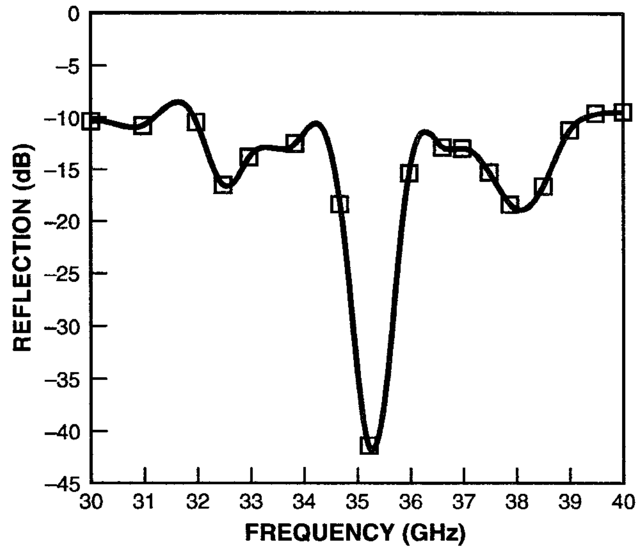


Fig. 11. Measured frequency dependence of the reflected power of a transmission-type rotator at angle of incidence $\theta_o = 45°$.

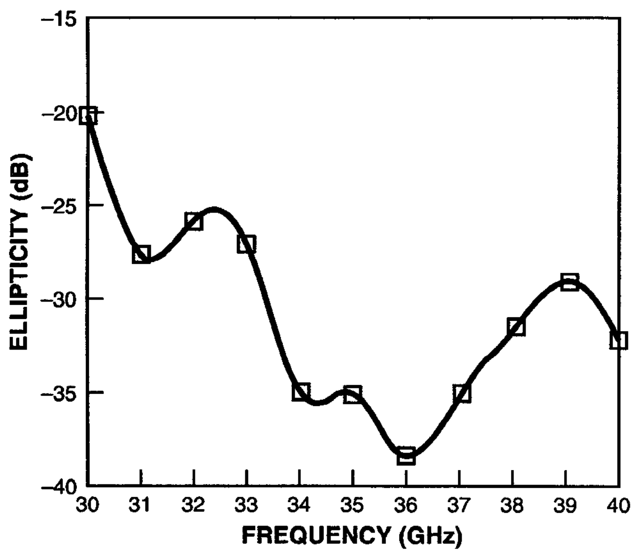


Fig. 10. Measured frequency dependence of the ellipticity of a transmission-type rotator at angle of incidence $\theta_o = 45°$.

## V. Experimental Results

Experiments have been performed on a transmission device at oblique incidence to compare with theory. There are internal reflections within the multilayer structure which exaggerate the Fabry–Perot effect. The Faraday rotation over a broad frequency range centered at ~35 GHz is shown in Fig. 9. The parameters for this device are as follows: the magnetization of the ferrite $4\pi M = 680$ Oe, the dielectric constant of the quartz is $\epsilon_d = 3.77$, that of the ferrite $\epsilon_f = 14.7$, the thickness of the ferrite is $T_f = 10$ mm, and that of the quartz $T_d = 1.2$ mm. The Faraday rotation deviates from the mean value by $\pm 2°$ over a 10-GHz bandwidth, slightly greater than that expected if the optimum parameters were chosen. Fig. 10 shows a plot of the ellipticity, again exhibiting higher values at the band edges, but still negligibly small. Although the parameters are not ideal, these results are in good agreement with theory, when the above parameters are used to evaluate the components of the scattering matrix. Fig. 11 shows the measured reflection loss of this experimental rotator.

## VI. Conclusions

This paper has outlined the procedure for treating plane-wave propagation with oblique incidence through a complex multilayer structure consisting of scalar dielectric and tensor magnetic media using a matrix formalism adapted from that developed by Zak *et al.* [5]. The process permits the evaluation of such pertinent quantities as the transmission, reflection, Faraday and Kerr coefficients, and the ellipticity. We have applied this technique to analyze both a transmission and a reflection circulator to deduce the insertion losses and the isolation. The reflection circulator, which permits surface cooling and is therefore capable of handling substantially higher average power, is nearly comparable in microwave performance to that of the transmission device at normal incidence. We have shown that the ferrite layer is inherently broadband and that the limitations on bandwidth are imposed by the quarter-wave plates required for matching. The analysis treated a single antireflection layer at oblique incidence. However, it is known from the optical analog that a two-layer structure would improve the performance further. We are considering this possibility for our millimeter-wave device.

For the present, we have also neglected the dissipative losses in the dielectric and ferrite media (conduction loss is incorporated in the case of the reflecting surface), but they can be included by using appropriate complex values for the dielectric constants and the magnetic susceptibilities in the components of the scattering matrix. From previous laboratory experience with the transmission version of the circulator, we are confident that, although these may add slightly to the insertion loss, they will not affect the isolation or the Faraday rotation significantly. The latter were the main objectives for design considerations of this paper. The final conclusion is that the reflection circulator will satisfy the requirements for a broadband device, but at significantly higher power than its transmission counterpart.

## Appendix I

The matrices in (5), which we shall designate as Zak matrices, have been defined as follows. First, the tangential components of the electric and magnetic fields $E_x$, $E_y$, $H_x$, $H_y$ at the interface are related to the amplitudes $E_s^{(i)}$, $E_p^{(i)}$, $E_s^{(r)}$, $E_p^{(r)}$ of the components, polarized $s$ (perpendicular) and $p$ (parallel) to the plane of incidence, of the incident ($i$) and reflected ($r$) waves. These sets of field components can be represented by single-column matrices $F$ and $P$, respectively [5, eq. (1)]

$$F = \begin{pmatrix} E_x \\ E_y \\ H_x \\ H_y \end{pmatrix}, \quad P = \begin{pmatrix} E_s^{(i)} \\ E_p^{(i)} \\ E_s^{(r)} \\ E_p^{(r)} \end{pmatrix}$$

A matrix connecting $F$ and $P$, called the medium boundary matrix $A$, is defined by the relation $F = AP$. This boundary matrix has been derived in [5] for the general case of the nonreciprocal birefringent magneto-optical medium, taking the following form in their notation [5, eq. (30)]:

$$A^{(\mathrm{POL})} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ \frac{i}{2}\alpha_y^2 Q & \alpha_z & \frac{i}{2}\alpha_y^2 Q & -\alpha_z \\ \frac{i}{2}\alpha_z QN & -N & -\frac{i}{2}\alpha_z QN & -N \\ \alpha_z N & \frac{i}{2}QN & -\alpha_z N & \frac{i}{2}QN \end{bmatrix}.$$

The superscript (POL) signifies a Faraday-type configuration in which the direction of magnetization is perpendicular to the plane of the ferrite slab. $\alpha_z = \cos\theta_f$ and $\alpha_y = \sin\phi_y$, where $\phi_f$ is the oblique angle of propagation in the ferrite, $N$ is the index of refraction, and $Q$ is equivalent to our $\kappa$, the off-diagonal component of the tensor susceptibility. For a dielectric medium: in $A^{(\mathrm{POL})}$ above, set $Q$ equal to zero. The inverse $(A^{(\mathrm{POL})})^{-1}$ is [5, eq. (D.4)]

$$(A^{(\mathrm{POL})})^{-1} = \frac{1}{2}\begin{bmatrix} 1 & 0 & \frac{i}{2}\frac{Q}{\alpha_z N} & \frac{1}{\alpha_z N} \\ -\frac{i}{2}\frac{\alpha_y^2}{\alpha_z}Q & \frac{1}{\alpha_z} & -\frac{1}{N} & \frac{i}{2}\frac{Q}{N} \\ 1 & 0 & -\frac{i}{2}\frac{Q}{\alpha_z N} & -\frac{1}{\alpha_z N} \\ \frac{i}{2}\frac{\alpha_y^2}{\alpha_z}Q & -\frac{1}{\alpha_z} & -\frac{1}{N} & \frac{i}{2}\frac{Q}{N} \end{bmatrix}.$$

The transfer matrix $D$ for the ferrite medium is [5, eq. (41)]

$$D = \begin{pmatrix} U\cos\sigma & U\sin\sigma & 0 & 0 \\ -U\sin\sigma & U\cos\sigma & 0 & 0 \\ 0 & 0 & U^{-1}\cos\sigma & +U^{-1}\sin\sigma \\ 0 & 0 & -U^{-1}\sin\sigma & U^{-1}\cos\sigma \end{pmatrix}$$

where $U = \exp[-i(2\pi/\lambda)N\alpha_z d]$ represents translation across the ferrite layer at wavelength $\lambda$, and the angle $\sigma$ accounts for Faraday rotation of the polarization: $\sigma = (\pi/\lambda)NQd$. For the dielectric medium: in $D$ above, $\sigma$ is set equal to zero.

## Appendix II

### *A. Extraction of the Scattering Coefficients from the Resultant Matrix S*

As discussed in Section III, the matrix $S$, the equivalent to that designated $M$ in [5], is the ordered product of the $4 \times 4$ boundary matrices $A_m$ (and their inverses) with medium propagation matrices $D_m$ for the succession of refracting layers $m = 1, 2, \cdots, m_o$:

$$S = A_0^{-1}\left(\prod_{m=1}^{m_o} A_m \overline{D}_m A_m^{-1}\right) A_{fin}.$$

The matrix $A_m$ embodies the relations connecting the amplitudes of the plane waves propagating within the $m$th layer, toward and away from its interface with the $(m-1)$th layer, in the $s$ and $p$ polarizations, and the components of the fields $\boldsymbol{E}$ and $\boldsymbol{H}$ tangential to the interface, which are subject to Maxwell boundary conditions. The matrix $D_m$ embodies the propagation vector characterizing the dielectric/magnetic properties of the $m$th medium as well as the direction of propagation (in accordance with Snell's law). In the case of a medium with gyrotropic properties, $D_m$ manifests the coupling of the $s$ and $p$ polarizations due to the Faraday effect. The initial and final boundary matrices $A_o$ and $A_{fin}$ relate to the media surrounding the $m_o$-layer structure.

As shown in [5], the resultant reflection and transmission coefficients for the multilayer structure are contained in the matrix $S$ as follows: partition $S$ into $2 \times 2$ submatrices according to

$$S = \begin{bmatrix} G & H \\ I & J \end{bmatrix}.$$

Then

$$G^{-1} = \begin{bmatrix} t_{ss} & t_{sp} \\ t_{ps} & t_{pp} \end{bmatrix} \quad \text{and} \quad IG^{-1} = \begin{bmatrix} r_{ss} & r_{sp} \\ r_{ps} & r_{pp} \end{bmatrix}$$

where $t_{ij}$, $r_{ij}$ are the resultant transmission and reflection coefficients (ratios of complex amplitudes of the emerging waves to that of the incident signal, referred to the polarization basis $s$, $p$); $ij$ take the identities $s$ or $p$.

For computational modeling of the reflection rotator, we identify $m = 0, 1, 2, 3$ with (0) empty space, (1) quartz quarter-wave plate, (2) axially magnetized ferrite, and (3) high-conductivity reflecting/cooling plate (see Fig. 1). A representative list of ten independent parameters required to characterize the structure is presented in Table I. With this or an equivalent set of parameters, all the elements of the matrices required for a complete model of the reflection-type rotator (eight matrix factors) or the transmission-type surrogate (with oblique incidence—eleven matrix factors) are determined. See Appendix I.

### B. Interpretation of the Scattering Coefficients in Terms of the Experimental Quantities Faraday Rotation Angle $\Theta_F$ and Ellipticity $\mathcal{E}$

Consider, for example, a signal selected for incidence on the front of the transmission rotator with amplitude $A$ (and reference phase value of zero), oblique angle of incidence $\theta_o$, and polarization $s$. The wave emerging from the rear surface is represented relative to $A$ by (complex) transmission coefficients $t_{ss}$ and $t_{ps}$ which by their amplitudes and phases characterize in general elliptical polarization with major principal axis oriented at the Faraday-rotation angle $\Theta_F$ relative to the axes of $s$ and $p$, and with (complex) amplitudes $E_{\max}$ and $E_{\min}$ at the orientations of the principal axes. We adopt the following definition of ellipticity:

$$\mathcal{E} = \frac{|E_{\min}|^2}{|E_{\max}|^2}.$$

To predict $\Theta_F$ and $\mathcal{E}$ from computed $t_{ss}$ and $t_{ps}$, let

$$\tan R = \frac{|t_{ps}|}{|t_{ss}|}, \qquad \delta = \frac{1}{2}\left[\arg(t_{ss}) - \arg(t_{ps})\right]$$

where $t_{ss} = |t_{ss}| \exp[i \arg(t_{ss})]$, etc. Then the angle $\Theta_F$ is given by

$$\tan 2\Theta_F = \tan 2R \cos 2\delta$$

and the ellipticity $\mathcal{E}$ by

$$\mathcal{E} = \frac{1 - \sqrt{1 - \sin^2 2R \sin^2 2\delta}}{1 + \sqrt{1 - \sin^2 2R \sin^2 2\delta}}.$$

To apply the above analysis in the case in which the incident signal is polarized $p$ rather than $s$, replace the transmission coefficients $t_{ss}$ and $t_{ps}$ by $t_{pp}$ and $t_{sp}$, respectively. Polarization states intermediate between $p$ and $s$ may of course be analyzed similarly, by application of the superposition principle.

TABLE I

| | | |
|---|---|---|
| 1 | Frequency $v$ | $30.0 \cdots 40.0$ GHz |
| 2 | Gyromagnetic ratio $\gamma/2\pi$ | $2.7992 \times 10^6$ Hz Oe$^{-1}$ |
| 3 | Ferrite $4\pi M_s$ | 1000 Oe |
| 4 | Angle of incidence $\theta_0$ | 45.0 deg |
| 5 | Thickness $d$ ($\lambda/4$ plate) $m = 1$ | 0.117 cm |
| 6 | (ferrite) $m = 2$ | 0.349 cm |
| 7 | Dielectric constant $\epsilon_r$, $m = 0$ | 1.00 |
| 8 | $m = 1$ | 3.83 |
| 9 | $m = 2$ | 14.68 |
| 10 | Resistivity $\rho$ (mirror) $m = 3$ | $1.72 \times 10^{-6}$ Ω cm |

In the case of the reflection rotator, the interpretation is similar, except that the transmission coefficients are replaced in the above analysis by the reflection coefficients $r_{ss}$ and $r_{ps}$ for $s$-polarized incident signal, or $r_{pp}$ and $r_{sp}$ for $p$-polarized.

### C. Determination of the Scattering Coefficients from Experimental Data, when Faraday-Rotation Angle $\Theta_F$ and Ellipticity $\mathcal{E}$ are Measured

Reversing the above reasoning, if $\Theta_F$ and $\mathcal{E}$ are known, the amplitude ratio $\tan R$ and phase difference $2\delta$ of the $s$- and $p$-polarization scattering coefficients may be determined as follows. Let

$$\rho_0 = \frac{\mathcal{E} - 1}{\mathcal{E} + 1}.$$

Then

$$\tan^2 R = \frac{1 + \rho_0 \cos 2\Theta_F}{1 - \rho_0 \cos 2\Theta_F}$$

and

$$\cos 2\delta = \frac{\tan 2\Theta_F}{\tan 2R}.$$

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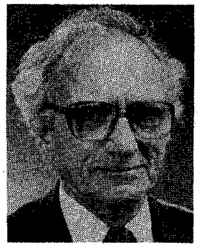

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